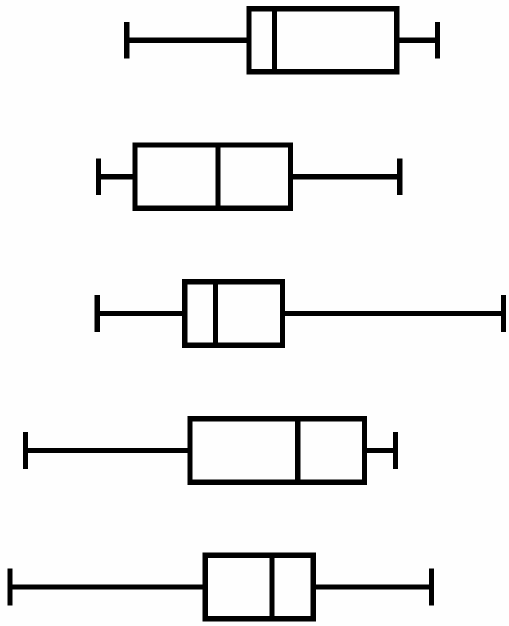
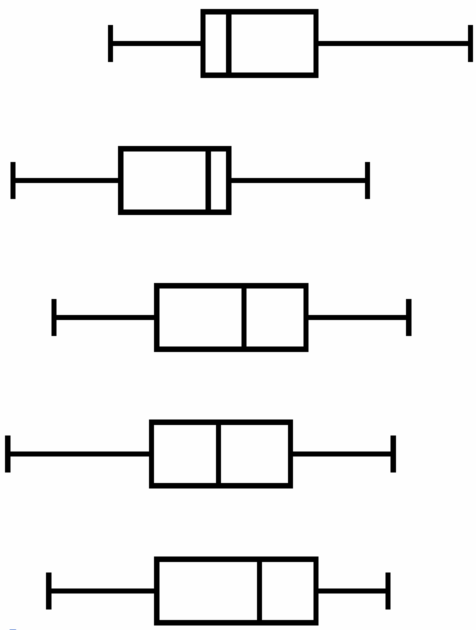


Northern koala

How much can a koala bear?



Prince Alfred College
Founded 1869

Noel Baker Centre for School Mathematics

CASIO®

How much can a koala bear?

Version 1.00 – August 2008

Written by Anthony Harradine and Alastair Lupton

Index

Section Page

Using this resource.

This resource is *not* a text book.

It contains material that is hoped will be covered as a dialogue between students and teacher and/or students and students.

You, as a teacher, must plan carefully 'your performance'. The inclusion of all the 'stuff' is to support:

- you (the teacher) in how to plan your performance – what questions to ask, when and so on,
- the student that may be absent,
- parents or tutors who may be unfamiliar with the way in which this approach unfolds.

Professional development sessions in how to deliver this approach are available.

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Legend.

EAT – Explore And Think.

These provide an opportunity for an insight into an activity from which mathematics will emerge – but don't pre-empt it, just explore and think!

At certain points the learning process should have generated some ***burning mathematical questions*** that should be discussed and pondered, and then answered as you learn more!

Time to Formalise.

These notes document the learning that has occurred to this point, using a degree of formal mathematical language and notation.

Examples.

Illustrations of the mathematics at hand, used to answer questions.

Background Reading.

NATIONAL GEOGRAPHIC NEWS
NATIONALGEOGRAPHIC.COM/NEWS

Koalas Overrunning Australia Island "Ark"

Bijal Trivedi
National Geographic News

May 10, 2002

On Kangaroo Island, a rugged eco-tourism paradise 30 minutes by air southwest of Adelaide, the koala is eating its way out of house and home and eucalyptus tree.
Wildlife officials are sterilizing and exporting the fuzzy little marsupials to the mainland, but not everybody thinks the population-control program makes ecological sense.

"Koalas are not the root but only a small part of the problem," maintains Deborah Tabart, executive director of the Australian Koala Foundation, a private research and conservation organization. She considers the government's random sterilization a "knee-jerk" response that disrupts the koala community.

Koalas, declared an endangered species by the U.S. Fish and Wildlife Service two years ago, are anything but threatened on the 737-square-mile island where they strip the manna gum eucalyptus trees they favour, eventually killing the trees. The government has sterilized about 3,400 adult koalas since 1997 and shipped out over 1,000.

"When we remove the koalas, the trees recovered. It is a clear indication that the koalas are the problem," says Keith Twyford, Kangaroo Island regional manager for National Parks and Wildlife South Australia.

The island is home to plant and animal species found nowhere else, but the koala wasn't always among them. When the koala faced extinction because of over-hunting for its fur, the government introduced about 18 adult koalas from French Island in Victoria in the early 1920s.

"They saw Kangaroo Island as an Ark where the animals could live free from the pressures of development," Twyford explains.

With no natural enemies, the koalas thrived, spending up to 20 hours a day snoozing in the eucalyptus trees and the rest of the time eating them. A 2000-2001 survey estimated about 27,000 koalas on the island. The koala foundation believes the total number in the country is around 100,000, but other estimates go as high as 400,000.

Elsewhere in the nation, koala populations are declining, threatened by wood-chip production, foxes, dogs, automobiles and urbanization. But the solution is not as simple as moving koalas from where there are too many to where there are too few. Several sub-species are involved, and the Kangaroo Island variety is larger than those found in Queensland and New South Wales.

1. *Experiencing sampling variation.*


Southern koala

Imagine a similar situation to the one described in the previous article, happening on *Koala Island*. The concerned citizenry want to find out more about the health of their island's koala population.

A National Parks and Wildlife Officer is given the task of developing a good description of how the weight of the female koalas in the population currently varies.

What strategy is most likely to be used to find out more about the weights of the female koalas in this population?

What factors would be important to consider in the implementation of this strategy?

1.1 EAT 1

Your teacher will provide you with access to this koala 'population'.
From this population you are to choose 5 random samples of 15.
Once you have each sample of 15, return the koalas and choose again.

Record the weights of each of your samples of 15 in the form of a dot plot (with big, clear dots) on the axes provided on page 15.

Will You Look At That! (WYLAT!)

Once your samples are all displayed, answer the following questions.

1. What do you notice about the samples?
2. Did they turn out as expected?
3. Make a summary of each sample so that you can quickly but accurately describe to others the characteristics of each of your samples (without showing them the dot plots).
4. What do you think these samples tell you about the population?

1.1 EAT 1 continued

With the help of your teacher, combine your dot plots with the others produced by the members of your class.

Will You Look At That! (WYLAT!)

Based your previous exploration, you should now have a greater understanding of the idea that samples randomly drawn from a population will *vary*.

5. Describe exactly which attributes have varied in the random samples that you have looked at.
6. To what degree is this variation *describable*?

2. *Looking closer at sampling variation.*

To gain a deeper understanding of how the mean of random samples drawn from a population will vary we need to draw lots of samples and fast!

To do this we will simulate a population of Northern Koalas (the somewhat cuter cousins of the Southern Koalas on Koala Island) with females that weigh between 3.6 and 6.6 kilograms, with an average of 5.1 grams. From our simulated population we can draw repeated samples and study how their means vary.

Northern koala

2.1 EAT 2

Run the program `NORMPOP` in [PRGM] of your CASIO *fx-9860G AU*.

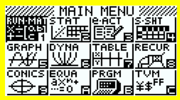
Enter the population mean μ and standard deviation σ that will simulate the weights of a population of female Northern Koalas.

Go to [STAT] and have a look at the population of 999 koalas you have created.

Sketch a representation of the weight distribution of the population of koalas you will be sampling from on the axis on page 16.

Run the program `REPSAMP` in [PRGM] of your CASIO *fx-9860G AU*, and thus take 30 random samples of size *n* (as instructed by your teacher).

Draw a single dot below the axis on page 16 to represent the first sample's mean. For each subsequent sample, make a dot (lower than the one before) to make a 'string of sample means'.



7.1, 7.2, 7.3

Based your previous exploration, you should now have a picture of the way that sample means vary, for your sample size *n*.

1. Describe how that sample means varied for your 30 samples of size *n*.

2.1 EAT 2 continued

With the help of your teacher, combine your picture of the way that sample means vary with the pictures generated by your classmates. Do so in a systemic fashion.

Will You Look At That! (WYLAT!)

2. Describe how the sample means varied in the 30 samples of size *n* that your classmates have studied.

3. What conclusion does this suggest?

3. *Conclusions about sampling variation.*

Coming out of Section 2, you should have an appreciation for the following:

From looking at your 30 sample means, the sample means varied, somewhat symmetrically, around the value of the mean μ of the population from which the samples were drawn.

From looking at the 30 sample means of your classmates in totality, the variation in sample means seems to be affected by sample size *n*. In particular, the larger the sample size *n*, the smaller the variation in sample means.

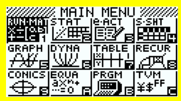
By looking closer at sample means we should be able to refine these ideas further.

3.1 EAT 3

Run the program `SEEMEANS` in PRGM of your CASIO *fx-9860G AU*.

Take 100 samples of the same size you used previously.

Look at the data now in `LIST 2`.



7.4

Formalisation – Read carefully this description of the situation at hand.

Given we have a population containing 999 elements and have the measure of one attribute (e.g. weight) of each element, denoted by the variable X we can say that X is distributed normally with mean (μ) = 5.1 and standard deviation (σ) = 0.5.

From this population we take 100 SRS (simple random samples) of a given size (n) and then compute and store the means for each. Let $\overline{X}_n$ be the mean of one SRS of size n. Therefore, given that we take 100 samples, we will have a distribution for $\overline{X}_n$ that will have a mean $\mu_{\overline{X}_n}$ and a standard deviation $\sigma_{\overline{X}_n}$.

1. Based on these definitions express, using notation, the relationships between μ, σ, $\mu_{\overline{X}_n}$ and $\sigma_{\overline{X}_n}$ that is described in words at the top of this page.

2. Alone, or with guidance from your teacher, use the program SEEMEANS to complete the following table,

Variable	**Shape of Distribution**	$\mu_{\overline{X}_n}$	$\sigma_{\overline{X}_n}$
$\overline{X}_4$			
$\overline{X}_8$			
$\overline{X}_{16}$			
$\overline{X}_{24}$			
$\overline{X}_{48}$			
$\overline{X}_{80}$			

3. What patterns are observable in this table?

4. See if you can develop a formula that would enable you to predict $\mu_{\overline{X}_n}$ and $\sigma_{\overline{X}_n}$, given μ and σ and a specific sample size n.

4. Consideration of non-normal populations.

Based on your study of the a number of distributions of sample means $\overline{X}_n$ (with differing n values), and in particular your analysis of $\mu_{\overline{X}_n}$ and $\sigma_{\overline{X}_n}$ you should have come up with the conjectures that,

For a normally distributed population X, with mean μ and standard deviation σ, from which simple random samples of size n are taken, the sample means $\overline{X}_n$ are normally distributed with

$$\mu_{\overline{X}_n} = \mu \text{ and } \sigma_{\overline{X}_n} = \frac{\sigma}{\sqrt{n}}$$

What questions does this raise?

4.1 EAT 4

Either alone or as instructed by your teacher, use the programs SKEWPOP1, SKEWPOP2 and SKEWPOP3 in conjunction with SEEMEANS to complete three versions of the table below (one for SKEWPOP1, one for SKEWPOP2 and one for SKEWPOP3).

7.2, 7.4, 7.5

SKEWPOP (...)	**Shape of Distribution**	μ	σ
X			
Variable	**Shape of Distribution**	$\mu_{\overline{X}_n}$	$\sigma_{\overline{X}_n}$
$\overline{X}_4$			
$\overline{X}_8$			
$\overline{X}_{16}$			
$\overline{X}_{24}$			
$\overline{X}_{48}$			
$\overline{X}_{80}$			

Describe your findings, based on this investigation.

5. *Application of the Central Limit Theorem.*

Based on your learning in this unit so far, in conjunction with your ability to perform calculations with normal distributions, you should be able to answer the following questions.

1. a. Write down the definition of the Central Limit Theorem as it is stated in a mathematics text that is relevant to your present course of study.
   b. Comment on or critique this definition in light of what you have learned from this unit of work.

2. If the reading comprehension score of a student in a year 8 class could be represented by X, a normally distributed random variable with a mean of $\mu = 80$ and standard deviation $\sigma = 20$, describe the distribution that would represent the mean reading comprehension score of a randomly chosen group of 4 such students.

3. If the weights of individual packets of 2 minute noodles varies according to a normal distribution with a mean of 85.8 grams and a standard deviation of 1.9 grams
   a. Describe the distribution that will describe the mean weight of a simple random sample of 5 such packets of noodles.

   Such a sample of 5 packets of noodles in a multi-pack which is labelled with "average weight of contents: 85 grams".
   b. Determine the proportion of such multi-packs with an average weight within 1 gram of the claimed average weight.
   c. Determine the proportion of such multi-packs with an average weight less than what is claimed.

4. If the weights of the male workers in an office complex can be modelled by X, a normally distributed variable with a mean of 81.2 kg and a standard deviation of 9.3 kg
   a. Describe the distribution of the means of simple random samples of 12 of these workers.
   b. If such random sample occupied an office lift, determine the probability that the combined weight of the occupants of the lift exceeds 1050 kg?

5. Based on long term study, it is known that the completion time related to the performance of a particular assembly line task varies, and that it has a mean of 93 seconds and a standard deviation of 11 seconds.

   a. Assuming that these times vary according to a normal distribution,

      i. Find the proportion of random selections of 10 such tasks that will have mean completion time of more than 100 seconds.

      ii. Find the proportion of random selections of 300 such tasks that will have mean completion time of more than 100 seconds.

   b. If, in fact, the completion time varied according to a distribution that showed a skew to the right, in what way would your answers to part (a) differ? Comment.

6. If the length of retaining wall bricks is distributed normally with a mean of 220 mm and a standard deviation of 0.8 mm,

   a. Describe the sampling distribution associated with laying 32 of such bricks.

   b. What assumption must be made before calculations can be made about the length associated with laying 32 of these bricks end to end?

   c. What is the probability that 32 of these bricks laid end to end would be less than 7 metres?

7. Premium Instant Coffee is produced in sachets. The net weight of the coffee in a randomly chosen sachet can be modelled by *W*, a normally distributed random variable with a mean of μ = 5 6 grams and a standard deviation of σ = 0 2 grams.

   a. Sketch the distribution of W on a labelled axis.

   These coffee sachets are sold in packs of twenty. Let W_{20} be the average of the net weights of the sachets in a randomly chosen pack.

   b. i. Write down the mean and the standard deviation of the distribution of W_{20}.

      ii. On your labelled axis, add a sketch of the distribution of W_{20}.

   The packs of twenty coffee sachets are labelled as containing 110 grams net.

   c. Find the probability that a randomly chosen pack of twenty sachets will contain less than its labelled weight.

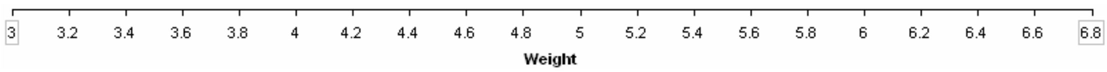
   The coffee sachets are also sold in bulk in catering boxes of 240 sachets.

   d. i. If 0.1% of catering boxes contain less than *k* grams, find *k* to the nearest whole gram.

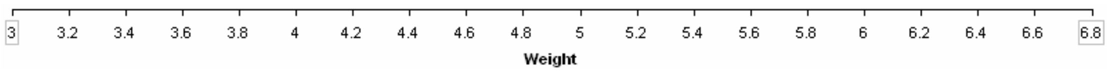
      ii. Would it be appropriate to label the catering boxes as containing 1.3 kilograms net? Give a reason for your answer.

6. *Appendix.*

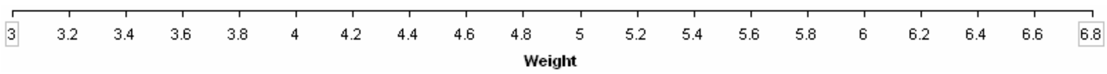
Sample 1



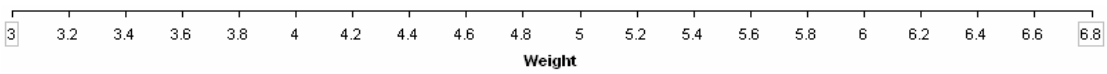
Sample 2



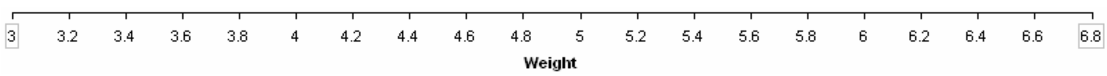
Sample 3

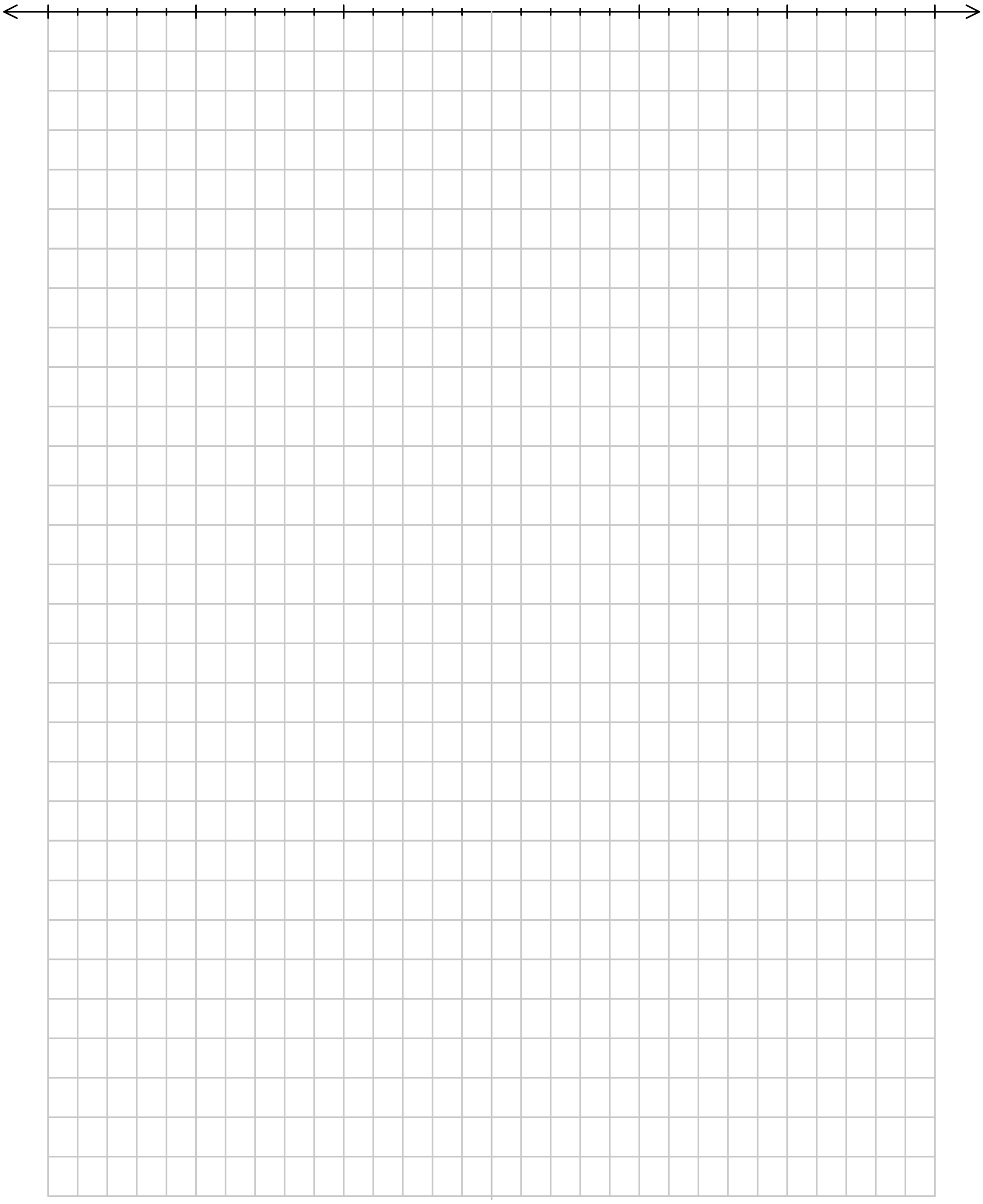


Sample 4



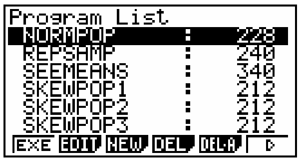
Sample 5





7. *eTech Support.*

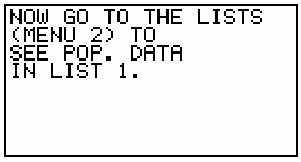
7.1 Running a program on the CASIO *fx-9860G AU* – NORMPOP.



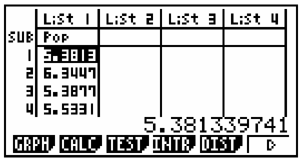
Go to PRGM, move the selection bar to NORMPOP and press EXE.

Enter the mean and standard deviation, followed by EXE.

Wait a few moments and the program will have placed in List 1 of STAT 999 normally distributed values with the mean and standard deviation you chose.



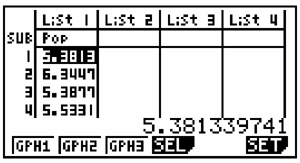
7.2 Drawing a Histogram of List data.

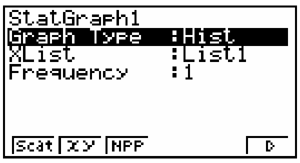


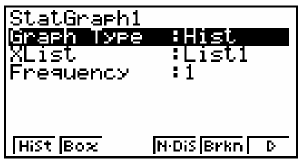
Go to STAT and you should see something like

To draw a Histogram of the data in List 1 (or another of the 26 available lists), first enter the Graph sub-menu by pressing GRPH F1

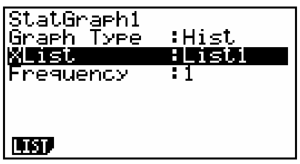
Press SET F6 to set the graph as a histogram of your chosen list.
To chose Histogram as your graph type, move the selection bar down to Graph Type, press ▷ F6 and then HIST F1 (see below).

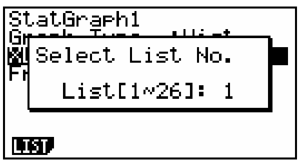






To change the XList – the list that will be represented in your Histogram - move the selection bar down to XLIST, press LIST F1 and then enter the number of the list to be represented, i.e. 1, followed by EXE.

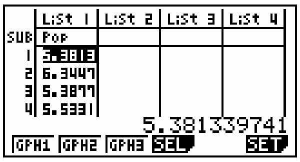


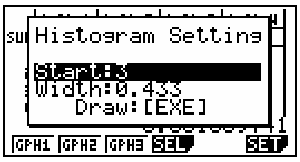


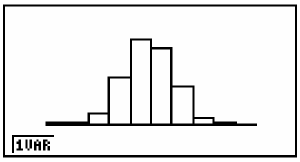
Once the Histogram is set up appropriately, it can be drawn by pressing `GPH1` [F1] whilst in the Graph sub-menu (below left).

A pop-up (like the one below centre) will show you the 9860's choice starting value and bar width for the histogram. These can be modified is desired, but if a quick 'snap shot' of the data is required it is not necessary to do so.

Press [EXE] to accept the settings and your histogram will be drawn.



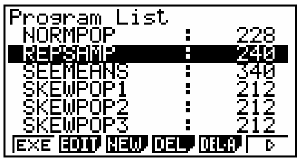




7.3 Running a program on the CASIO *fx-9860G AU* – `REPSAMP`.

> ***NB:*** A program like `NORMPOP` or `SKEWPOPX` must be run,
> generating a population in `List 1` of STAT, before this sampling program can run.

Go to PRGM, move the selection bar to `REPSAMP` and press [EXE]

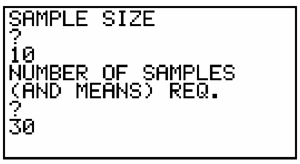


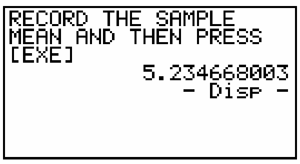
Enter the required sample size and number of samples, followed by [EXE] (below left).

Record each mean as instructed and then press [EXE] to obtain the next mean.

At the end of the chosen number of samples the program will say `Done`.

By pressing [EXE] you will then be returned to the `Program List` screen

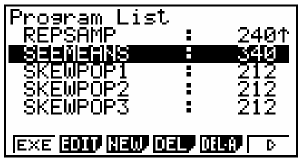




7.4 Running a program on the CASIO *fx-9860G AU* – SEEMEANS.

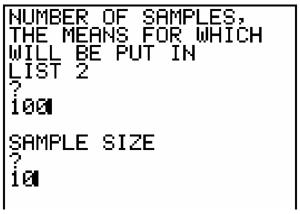
> ***NB:*** A program like NORMPOP or SKEWPOPX must be run, generating a population in List 1 of STAT, before this sampling program can run.

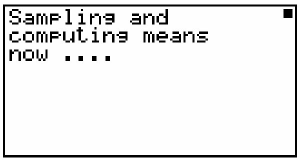
Go to PRGM, move the selection bar to SEEMEANS and press EXE



Enter the number of samples required and the sample size, followed by EXE (below left).

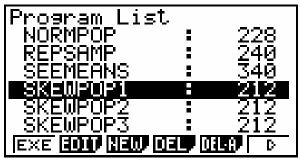
Wait a few moments and the samples requested will have been taken and their means computed and stored in List 2 in STAT.



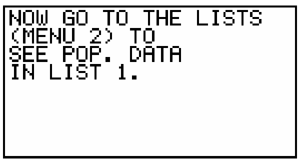


7.5 Running a program on the CASIO *fx-9860G AU* – SKEWPOPX.

Go to PRGM, move the selection bar to SKEWPOP1 and press EXE



This will, after a few moments wait, have placed in List 1 of STAT a skewed population made up of 999 elements.



NB:

The nature of the skewed population varies between SKEWPOP1, 2 and 3.

Each population needs to be generated and studied (using SEEMEANS) one at a time (i.e SKEWPOP1 then SKEWPOP2…) as the use of each program overwrites the contents of List 1 of STAT.

8. *Answers.*

5. Application of the C.L.T.

1.

"For random samples from a population which is not necessarily normally distributed, the distribution of sample means is approximately normal when the sample size *n* is large. As *n* increases the approximation to a normal distribution improves."

The definition of 'large' in relation to sample size is sometimes given as a specific value i.e. "*n*=30 is large"

As we have seen, such a value is more than adequate when sampling from some distributions but inadequate when sampling from heavily skewed distributions.

Decisions about what constitutes a large enough sample must be made on a case by case basis, with some thought to the skew/asymmetry of the distribution to be sampled from.

2.

$\overline{X}_4$ would be normally distributed with a mean of 80 and a standard deviation of $\frac{20}{\sqrt{4}} = 10$.

3. (a)

The mean weight $\overline{X}_5$ is normally distributed with a mean of 85.8 kg and a standard deviation of $\frac{1.9}{\sqrt{5}} \approx 0.85$.

3. (b)

$P(84 \le \overline{X}_5 \le 86) = 0.576$

3. (c)

$P(\overline{X}_5 \le 85) = 0.173$

4. (a)

The distribution of the means of samples of size 12 will be normal is shape with a mean of 81.2 kg and a standard deviation of 2.68 kg (to 3 sig. fig.'s).

4. (b)

$\frac{1050}{12} = 87.5$ kg

$P(\overline{X}_{12} \ge 87.5) = 0.00947$

5. (a) (i)

Using a normal distribution with $\mu = 93$ and $\sigma = \frac{11}{\sqrt{10}} \approx 3.48$

$P(\overline{X}_{10} \ge 100) = 0.022$.

5. (a) (ii)

Using a normal distribution with $\mu = 93$ and $\sigma = \frac{11}{\sqrt{300}} \approx 0.635$

$P(\overline{X}_{300} \ge 100) = 1.49 \times 10^{-28}$

5. (b)

Depending on the degree of skew, the answer to (a) (i) may be wrong (and an underestimate), as the distribution of the means of samples of size 10 may not be normally distributed.

Due to the larger sample size, the answer to (a) (ii) is less likely to be wrong and in most circumstances could be relied upon.

6. (a)

The mean length $\overline{X}_{32}$ is normally distributed with a mean of 220 mm and a st. dev. of $\frac{0.8}{\sqrt{32}} \approx 0.141$ mm.

6. (b)

We must assume that the bricks to be laid constitute a simple random sample.

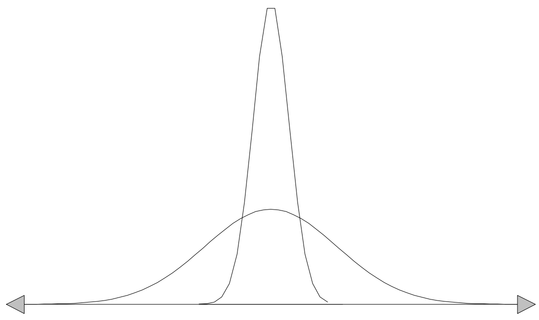
6. (c)

$\frac{7000}{32} = 218.75$ mm

$P(\overline{X}_{32} \le 21.875)$

$= 4.84 \times 10^{-19}$

7. (a)



7. (b) (i)

$\mu_{\overline{X}_{20}} = 5.6$

$\sigma_{\overline{X}_{20}} = \frac{0.2}{\sqrt{20}} = 0.0447$

7. (b) (i)

See Above

7. (c)

$\frac{110}{20} = 5.5$ grams

$P(\overline{X}_{20} \le 5.5) = 0.0127$

7. (d) (i)

As $\mu_{\overline{X}_{240}} = 5.6$ and

$\sigma_{\overline{X}_{240}} = \frac{0.2}{\sqrt{240}} = 0.0129$,

$P(\overline{X}_{240} \le k) = 0.001$

$k = 5.56$

7. (d) (i)

$5.56 \times 240 = 1334$

As 99.9% of caterers boxes will contain more than 1334 grams, a claim of 1300 grams (1.3 kg) is appropriate (if a little conservative).